7. (a) Determine the analytic function whose real part is $e^x[x\cos y - y\sin y]$.

(b) Prove that every Mobius transformation maps circles or straight lines into circles or straight lines.

Unit-IV

8. (a) Evaluate $\oint \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}dz$, the where C is the circle $|z| = 3$.

(b) Find the Laurent's series expansion of $\frac{z^2-2}{z^2+5z+6}$ in the region

(i) $2 < |z| < 3$ (ii) $|z| > 3$

9. (a) Evaluate the integral $\int_0^{\pi}\frac{\cos\theta}{3+\sin\theta}d\theta$, by contour integration.

(b) $\oint_c \frac{4-3z}{z(z-1)(z-2)}dz$ where, C is the circle $|z|=\frac{3}{2}$.

B.Tech. (M.E.) 2nd Semester

(G-Scheme) Examination, May-2023

MATH - II

Paper - B.Sc.-MATH-102-G

Multivariable Calculus, Differential Equations and Complex Analysis

Time allowed : 3 hours] *[Maximum marks : 75*

Note : ***Question no. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal marks.***

1. (a) Show that, $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$.

(b) Solve: $2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$.

(c) Solve: $\frac{d^2y}{dx^2} + y = 0$.

(d) Split up into real and imaginary parts e^{e^z}.

(e) State the necessary and sufficient conditions for a function to be analytic.

(f) Define Mobius transformations. 6×2.5

Unit-I

2. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz dy dx$.

(b) Evaluate by changing the order of integration

$$\int_0^{\infty} \int_0^{x} x.e^{-\frac{x^2}{y}}.dy\, dx.$$

3. (a) If $\vec{F} = (3x^2 + 6y)\hat{i} - 14\, yz\hat{j} + 20\, xz^2\hat{k}$, evaluate the line integral $\int_C \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$.

(b) Using divergence theorem for

$\vec{F} = (x^2 + yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$,

find $\int \vec{F}.d\vec{S}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

Unit - II

4. (a) Solve the equation,

$$\left[y\left(1+\frac{1}{x}\right) + \cos y\right]dx + (x + \log x - x\sin y)dy = 0.$$

(b) Solve : $\frac{dy}{dx} + \frac{1}{x}y = 3x^2y^3$.

5. (a) Find the power series solution of

$$(1 - x^2)\frac{d^2y}{dx^2} + 2y = 0, \text{ where } y(0) = 4 \text{ and } y'(0) = 5.$$

(b) Solve :

$$(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4\cos\log(1 + x).$$

Unit-III

6. (a) If $\tan(\theta + i\varphi) = \tan\alpha + i\sec\alpha$, Show that

(i) $e^{2\varphi} = \pm\cot\alpha/2$

(ii) $2\theta = (n + 1/2)\pi + \alpha$

(b) Prove that the functions defined by

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, z \ne 0 \text{ and } f(0) = 0$$

is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.