

(4)

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9. (a) Evaluate

10

(b) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ as Laurent's series about $z = -2$.

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B.Tech. (ME) 2nd Semester (G-Scheme)

Examination, July-2022

MATH-II

Multivariable Calculus, Differential equations and
Complex Analysis

Paper- BSC-(Math-102-G)

Time allowed : 3 hours]

[Maximum marks : 75

Note : Question number 1 is compulsory. Attempt five questions in total by selecting one form each unit. All questions carry equal marks.

1. (a) Evaluate $\int_3^4 \int_1^2 (xy + e^y) dy dx$

(b) Solve $(2xy + y - \tan x) dx + (x^2 - x \tan^2 y + \sec^2 y) dy = 0$

(c) Define Mobius transformation and its determinant.

(d) If $Z = x + iy$, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$

(e) Evaluate $\oint_C \frac{dz}{z-a}$, where C is circle $|z-a|=r$.

(f) If $f(z) = \frac{1+e^z}{\sin z + Z \cos z}$, find residue at $z=0$

$2.5 \times 6 = 15$

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3015-P-4-Q-9 (22)

[P.T.O.]

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Unit-I

2. (a) By changing the order, solve the integral

$$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$$
 7.5
- (b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$ by changing to polar co-ordinate. Hence deduce that

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$
 7.5
3. Verify Green's theorem in the plane for $\oint_C (xy + y^2) dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 15

Unit-II

4. (a) Solve $y'' - 2y' + 2y = e^x \tan x$ by using method of variation of parameters. 7.5
- (b) Solve the Cauchy-Euler equation. 7.5

$$x^2 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - 20y = (x+1)^2$$
5. (a) Express $4x^3 - 2x^2 - 3x + 8$ in the terms of Legendre's polynomial. 7.5
- (b) Find the power series solution of $\frac{d^2y}{dx^2} + x^2y = 0$ 7.5

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Unit-III

6. (a) Find the values of A and B such that the function $f(z) = x^2 + Ay^2 - 2 \times y + i(Bx^2 - y^2 + 2xy)$. 7.5
- (b) Determine the analytic function whose real part is $e^{2x} \times \cos 2y - e^{2x} y \sin 2y$. 7.5
7. (a) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express $u + iv$ as an analytic function of z . 7.5
- (b) Find the Mobius transformation that maps the points $z = -1, 0, 1$ in the z -plane onto the points $w = 0, i, 3i$ in the w -Plane. 7.5

Unit-IV

8. (a) Evaluate $\oint_C \frac{3z(z + \frac{1}{3})}{(z^2 - 1)(z + 2)} dz$, where C is the circle $|z - 1| = 1$. 8
- (b) Verify Cauchy's integral theorem by integrating e^{iz} along the boundary of the triangle with vertices at the points $1 + i, -1 + i$ and $-1 - i$. 7

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[P.T.O.]