(b) Apply convolution theorem find

$$L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right].$$ 10

7. (a) Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{s}\right)$.

(b) Solve the integral equation $\int_0^t \frac{y(u)}{\sqrt{t-u}}\,du = 1+t+t^3$.

2×10=20

Section-D

8. (a) Find the differential equation of all planes which are at a constant distance 'a' from the origin. 10

(b) Solve the partial differential equation

$$(x^2-y^2-z^2)p + 2xyq = 2xz.$$ 10

9. (a) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$.

(b) Use the method of separation of variable to solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$. 2×10=20

B.Tech. (Common for All Branches) 2nd Semester

(F Scheme) Examination, July–2022

MATHEMATICS-II

Paper–Math-102-F

Time allowed : 3 hours] *[Maximum marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : ***Question No. 1 is compulsory****. Attempt total five questions with selecting* ***one*** *question from each section. All questions carry equal marks.*

1. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla r^n = n^{n-2}\,\vec{r}$.

(b) Evaluate $\iint_S \vec{r}.\hat{n}\,ds$, where S is a closed surface.

(c) Solve $(\sec x.\tan x.\tan y - e^x)dx + \sec x \sec^2 y\,dy = 0$.

(d) Solve $xdy - ydx = xy^2dx$.

(e) Find the Laplace transform of 2^t.

(f) Find the inverse Laplace transform of $\frac{e^{-2s}}{s-1}$.

(g) Solve : $p^2 + p = q^2$.

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(h) Solve : $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that, when $x=0$, $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$. 8×2.5=20

Section-A

2. (a) Find the directional derivative of the function $f(x, y, z) = 2xy + z^2$ at the point (1, –1, 3) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 10

(b) If the vector $\vec{F} = (ax^2y + y)\hat{i} + y\hat{j} + z\hat{k}$,
Show that $div\ (grad\ r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. 10

3. (a) Use Green's theorem in a plane to evaluate the integral
$\oint_c [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary in xy-plane of area enclosed by the x-axis and the semi-circle $x^2+y^2=1$ in the upper half xy-plane.

(b) Find $\iint_S \vec{F}.\hat{n}\,ds$ where
$\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having centre at (3,–1,2) and radius 3. 2×10=20

Section-B

4. (a) Solve the differential equation
$(xy \sin xy + \cos xy)y dx + (xy \sin xy - \cos xy)x dy = 0$. 10

(b) If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes and the time when the temperature is 40°C. 10

5. (a) Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = \sin(\log x)$.

(b) An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 *ohms* and a condenser of capacitance 25 *micro-farads*. Find the charge q and the current i at any time t, given that at $t=0$, $q = 0.05$ *coulomb*, $i = \frac{dq}{dt} = 0$, when $t = 0$.

2×10=20

Section-C

6. (a) Find the Laplace transform of :
(i) $t \sin 2t \cos 3t$
(ii) $\int_0^\infty \frac{\cos 4t - \cos 2t}{t}\,dt$. 10