

(d) Evaluate : $\int_0^a \int_0^{a-x} \int_0^{a-x-y} dz dy dx$

(e) Solve $(x^2y^2 + xy + 1)y dx + (x^2y^2 - xy + 1)x dy = 0$.

(f) Prove $\frac{1}{D-a} X = e^{ax} \int X e^{-ax} dx$ where $D = \frac{d}{dx}$ and X is function of x . $6 \times 2\frac{1}{2} = 15$

UNIT - I

2. (a) If $u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$ find value of

$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. $7\frac{1}{2}$

(b) If $\theta = r^n e^{-r^2/4t}$, what value of n will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$? $7\frac{1}{2}$

3. (a) In a plane triangle, find the maximum value of $\cos A \cos B \cos C$. $7\frac{1}{2}$

(b) If $xyz = 8$, find the values of x, y for which $u = \frac{5xyz}{x+2y+4z}$ is a maximum. $7\frac{1}{2}$

UNIT - II

4. (a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ and hence evaluate the same. $7\frac{1}{2}$

(b) Transform $\int_0^\pi \int_0^a r^3 \sin \theta \cos \theta dr d\theta$ to Cartesian form and hence evaluate. $7\frac{1}{2}$

5. (a) Find the area lying inside the Cardioid $r = a(1 + \cos \theta)$ and outside the circle $r = a$. $7\frac{1}{2}$

(b) Evaluate : $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dz dx dy$ $7\frac{1}{2}$

UNIT - III

6. Solve the following differential equations : $7\frac{1}{2} + 7\frac{1}{2} = 15$

(a) $\frac{dy}{dx} + x \sin^2 y = x^3 \cos^2 y$

(b) $(1-x^2) \frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

7. (a) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C . What will be the temperature of the body after 40 minutes from the original? $7\frac{1}{2}$

(b) The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series, is

$$E = Ri + \int \frac{i dt}{C}$$

Find the current i at any time t when $E = E_m \sin \omega t$. $7\frac{1}{2}$

UNIT - IV

8. (a) Solve : $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ 7½

(b) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$
7½

9. (a) Solve : $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x)$ 7½

(b) Solve the simultaneous equations : 7½

$$\frac{dx}{dt} + 2y = e^t, \quad \frac{dy}{dt} - 2x = e^{-t}$$

Roll No.

3034

**B. Tech. 3rd Semester (CSE)
Examination – December, 2022**

**MATHEMATICS - III (MULTIVARIABLE CALCULUS AND
DIFFERENTIAL EQUATIONS)**

Paper : BSC-MATH-203-G

Time : Three Hours]

[Maximum Marks : 75

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Unit. Question No. 1 is compulsory. All questions carry equal marks.

1. (a) If $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial^2 r}{\partial x^2}$ and $\frac{\partial^2 \theta}{\partial x^2}$.

(b) If $u = f(r, s, t)$ and $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$, find value of

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$$

(c) Evaluate : $\int_0^1 \int_0^x e^{y/x} dy dx$