

UNIT – IV

8. (a) For what value of  $p$  and  $q$ , the following system of equations :

7.5

$$x + y + z = 6, x + 2y + pz = q, x + 2y + 3z = 10 \text{ have :}$$

(i) no solution

(ii) unique solution

(iii) many solutions

(b) Prove that inverse and transpose of an orthogonal matrix are orthogonal. 7.5

9. Find the Eigen values and Eigen vectors of the matrix : 15

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

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Roll No. ....

3007

**B. Tech. 1st Semester (ME)  
Examination – December, 2022  
MATHEMATICS - I (Calculus and Matrices)  
Paper : BSC-MATH-101-G**

Time : Three Hours ]

[ Maximum Marks : 75

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

**Note :** Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) Evaluate :  $\beta(2.5, 1.5) = ?$
- (b) Evaluate the integral :  $\int_0^1 \int_0^1 \frac{x^2 y^3}{2} e^{dy} dx$ .
- (c) Test the convergence of series :  $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \sqrt{\frac{3}{8}} + \sqrt{\frac{4}{10}} + \dots \dots \dots \infty$
- (d) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  then find  $\text{div } \vec{r} = ?$

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(e) Find the rank of matrix :  $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$ .

(f) Using Cayley-Hamilton theorem find  $A^8$  where  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ . 2.5 × 6 = 15

### UNIT - I

2. (a) Examine the function :

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$$

for maxima and minima.

7.5

(b) For every value of  $x$ , show that :

7.5

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^{r-1} \frac{x^{2r-1}}{(2r-1)!} + (-1)^r \frac{x^{2r}}{(2r)!} \sin \theta x$$

where  $0 < \theta < 1$

3. (a) Find the volume of solid formed by the revolution about the  $x$ -axis of the loop of the curve : 7.5

$$y^2 = \frac{x^2(a+x)}{(a-x)}$$

(b) State and prove relation between  $\beta$ - $\gamma$  functions. 7.5

### UNIT - II

4. Test the convergence of the series : 7.5 + 7.5 = 15

(a)  $1 + \frac{1}{2} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots \dots \dots \infty, x > 0$

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(b)  $1 + x + \frac{1}{2} \frac{x^3}{3} + \frac{1.3 x^5}{2.4 \cdot 5} + \frac{1.3.5}{2.4.6} \frac{x^7}{7} + \dots \dots \dots x, x > 0$

5. (a) Expand  $\sin z$  as a Taylor series about  $z = \frac{\pi}{4}$ . 7.5

(b) If  $f(x) = \begin{cases} x & \text{for } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$ , then show that

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \left[ \frac{\cos 2x}{1^2} + \frac{\cos 6x}{3^2} + \frac{\cos 10x}{5^2} + \dots \dots \dots \right] \quad 7.5$$

### UNIT - III

6. (a) Discuss the continuity for :

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

at the point (0, 0). 7.5

(b) If  $u = x^y + y^x$  then using that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ . 7.5

7. (a) Examine for extreme values of :

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$$

(b) Find curl (curl  $\vec{v}$ ), where  $\vec{v} = (2xz^2)\hat{i} - yz\hat{j} + 3xz^3\hat{k}$  at (1, 1, 1). 7.5

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