7. (a) Find a linear transformation $T : R^4 \rightarrow R^3$ whose image is generated by (1, 2, 3) and (4, 5, 6). 6

(b) Let T be a linear operator defined by :

$T(x, y, z) = (2y + z, x - 4y, 3x)$.

Find the matrix of T w.r.t. the basis $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$. 6.5

UNIT – IV

8. Find the Eigen values and corresponding Eigen vector of the matrix : $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 12.5

9. (a) Verify that the matrix : $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal. 6

(b) If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ compute the product AB. 6.5

Roll No. ....

3008

B. Tech. 1st Semester (CSE) Examination – December, 2022

MATH - I (CALCULUS AND LINEAR ALGEBRA)

Paper : BSC-MATH-103-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is ***compulsory***. Marks are shown against each question.

1. (a) Evaluate : $\lim\limits_{n \to \infty} \dfrac{e^x - e^{\sin x}}{x - \sin x}$. 2.5

(b) Using Rolle's theorem for $f(x) = (x+2)^3(x-3)^4$ find the value of x in (–2, 3). 2.5

(c) Evaluate : $\int_0^1 \dfrac{1}{\sqrt{1 - x^4}} dx$ in terms of gamma function. 2.5

(d) Find the rank of the matrix : $\begin{bmatrix} 3 & 4 & 1 & 2 \\ 3 & 2 & 1 & 4 \end{bmatrix}$. 2.5

(e) Evaluate : $3A - 4B$, where $A = \begin{bmatrix} 3 & -4 & 6 \\ 5 & 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ 2.5

(f) Determine whether the following set of vectors (1, 1, 1), (0, 4, 1), (3, 0, 1) are linearly independent or linearly dependent. 2.5

(g) Write the zero vector in the vector space R^3 and R^4. 2.5

(h) Examine whether the following set of vectors forms a basis of R^2 : (0, 1), (0, –3). 2.5

(i) Find $T : R^2 \to R$ defined by $T(x, y) = xy$ is a linear transformation. 2.5

(j) Prove that $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is orthogonal. 2.5

UNIT – I

2. (a) Evaluate : $\lim_{x \to 1}\left[\frac{x}{x-1} - \frac{1}{\log x}\right]$ 6

(b) Using Taylor's theorem express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 1)$. 6.5

3. (a) Find the surface area of the solid formed by revolving the cardioids $r = a(1 + \cos\theta)$ about the initial line. 6

(b) Show that : $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1} dy = \frac{(p-1)!}{q^p}$ where $p > 0$, $q > 0$. 6.5

UNIT – II

4. (a) Solve the following system of equations : $2x - y + z = 3$; $x + 3y - 2z = 1$; $x + y + z = 6$ by Cramer's rule. 6

(b) If A and B are symmetric, prove that AB is symmetric iff AB commute. 6.5

5. (a) Find the rank of the matrix : $\begin{bmatrix} 3 & 4 & 1 & 2 \\ 3 & 2 & 1 & 4 \\ 7 & 6 & 2 & 5 \end{bmatrix}$. 6

(b) Solve the following equations : $2x + y + 4z = 12$; $8x - 3y + 2z = 20$; $4x + 11y - z = 33$ by Gauss Jordan method. 6.5

UNIT – III

6. (a) Is the set of all polynomials over R with constant term zero, form a vector space over reals ? If not why ? 6

(b) Find the basis and dimension of the vectors of R^4 generated by (1, –4, –2, 1), (1, –3, –1, 2), (3, –8, –2, 7). 6.5