Roll No. ....

OLE-3034

B. Tech. 3rd Sem. (CSE)
Examination – April, 2021

MATHEMATICS - III (Multivariable Calculus and Differential Equations)

Paper : BSC-MATH-203-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. **1** is ***compulsory***.

1. (a) If $v = \log(\tan x + \tan y + \tan z)$, find the value of :

$$\sin 2x \frac{\partial v}{\partial x} + \sin 2y \frac{dy}{dv} + \sin 2z \frac{\partial v}{\partial z}$$

(b) If $f(x, y) = \frac{xy^3}{x^2 + y^{6'}}$ then test $\lim_{\substack{x \to 0 \\ y \to 0}} f(x, y)$ exists or not.

(c) Evaluate $\int_0^1 \int_0^x (x+5)\, dy\, dx$.

(d) Solve $x\frac{dy}{dx}+y=xe^{x}$.

(e) Define exact differential equation. Write the necessary and sufficient condition for the first order differential equation to be exact.

(f) Find the particular integral of $\left(D^2-2D+1\right)y=\cosh hx$. 2.5 × 6 = 15

UNIT – I

2. (a) If $u=\operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$, prove that, 8

$$x^2\frac{\partial^2 u}{dx^2}+2xy\frac{\partial^2 u}{\partial x dy}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}\left(13+\tan^2 u\right)$$

(b) If $z(x+y)=x^2+y^2$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$. 7

3. (a) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. 8

(b) If $u=f(x,y), x=r\cos\theta, y=r\sin\theta$, then show that 7

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{dy}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

UNIT – II

4. (a) Evaluate $\int_0^{\frac{a}{\sqrt{2}}}\int_y^{\sqrt{a^2-y^2}} x\,dx\,dy$ by changing the order of integration. 8

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. 7

5. (a) Evaluate $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2 - y^2}{x^2 + y^2}\, dx\; dy$ by changing into polar coordinates. 8

(b) Find area enclosed by the leaves of the curve $r = a \sin 3\theta$. 7

UNIT – III

6. (a) Solve $\cos x \frac{dy}{dx} - y \sin x = y^3 \cos^2 x$. 8

(b) Solve $\left(x^2 + y^2 + 1\right)dx + x\left(x - 2y\right)dy = 0$. 7

7. (a) In a circuit containing inductance L, resistance R and voltage E, the current i is given by : 8

$$L\frac{di}{dt} + Ri = E$$

Given L = 640 henry, R = 250 ohms, E = 500 volts and at t = 0, i = 0, find the time that elapses before it reaches 90% of its maximum value.

(b) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter. 7

UNIT – IV

8. (a) Solve $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + 5y = x \cos(\log x) + 5$ 8

(b) Solve $\frac{d^2 y}{dx^2} - 9y = x \cos 2x$. 7

9. (a) Solve the following simultaneous differential equations : 8

$$\frac{dx}{dt} + 2x - 3y = t; \frac{dy}{dt} - 3x + 2y = e^{2t}$$

(b) Solve $\frac{d^2 y}{dx^2} + y$ = cot x coses x using method of variation of parameters. 7