Roll No. .....

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B. Tech. 3rd Semester (Civil Engg.) Examination – March, 2021

MATHEMATICS - III

Paper : BSC-MATH-205-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) Solve $p + q = pq$.

   (b) Solve $yp - 2yx = \log q$.

   (c) Explain working rule of trapezoidal to solve $\int_a^b f(x)dx$.

   (d) Find Laplace transform of $e^{-2t} \sin^2 3t$.

   (e) Find $L^{-1}\left(\frac{3(s^2-1)^2}{2s^5}\right) = ?$

(f) If $(G, \bullet)$ is group, then show that left inverse of an element is also its right inverse.

UNIT – I

2. (a) Solve $p^2y - qx = -q^2y$ by using Charpit's method.

   (b) Solve $(y^2 + z^2)p - 2xyq = x^2p - 2xz$.

3. (a) Solve $(x^2 - yz)p + xy = (zx - y^2)q + z^2$.

   (b) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function $u(x, t)$.

UNIT – II

4. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Trapezoidal rule and Simpshon's $\frac{1}{3}$ rule taking $h = 0.25$.

5. Use Regula Falsi method to compute the real root of the equation $x^3 - 9x + 1 = 0$.

UNIT – III

6. State and prove Convolution theorem and verify it for $\frac{1}{(s^2+1)(s^2+4)}$.

7. (a) Find $L^{-1}\left\{\frac{us+5}{(s-1)^2(s+2)}\right\} = ?$

   (b) Solve $y'' + 4y' + 7y = e^{-t}$, $y(0) = y'(0) = 1$, by using Laplace Transforms.

UNIT – IV

8. (a) (i) How many solutions are there of $x + y + z = 17$, subject to the constraints $x \geq 1$, $y \geq 2$ and $z \geq 3$.

   (ii) Show that if any five integers from 1 to 8 are chosen, then at least two of them will have sum 9.

   (b) State and prove Lagrange's Theorem.

9. (a) Prove a non-empty subset H of a group G is a subgroup of G if and only if :

   (i) $a \in H,\ b \in H \Rightarrow a.b \in H$

   (ii) $a \in H \Rightarrow \bar{a}^1 \in H$

   (b) If $(G, \bullet)$ is cyclic group generated by 'a' of order n, then a^m is a generator of G iff the g.c.d. of m and n is 1.
