(b) Define bilinear transformation. Find that bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. 7.5

UNIT – IV

8. (a) Evaluate the integral $\oint_C \frac{z}{(z-1)(z-3)} dz$; where C is the circle $|z| = \frac{z}{2}$. 7.5

(b) Expand $f(z) = \cos z$ in a Taylor series about the point $z = \frac{\pi}{4}$. 7.5

9. (a) Evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)} dz$, where C is the circle $|z| = 2$; using Residue Theorem. 7.5

(b) Evaluate : 7.5

$$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$$

Roll No. ....

3015

B. Tech. 2nd Semester (ME) Examination – July, 2021

MATH-II(Multivariable Calculus, Differential Equations & Complex Analysis)

Paper : BSC-MATH-102-G

Time : Three hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting ***one*** question from each Unit. Question No. **1** is ***mandatory***. All questions carry equal marks.

1. (a) Evaluate the following integral :

$$\left(\int_1^2 \int_1^3 xy^2 dx\, dy\right)$$

(b) State the result of Green's Theorem.

(c) Find the complementary function of differential equation :

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

(d) Solve the following Clairaut's equation :

$$p = \log(px - y)$$

(e) Separate real and imaginary parts of :

$$e^{(4+2i)^2}$$

(f) Find the constant 'p' such that $f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$ is analytic.

$6 \times 2.5 = 15$

UNIT – I

2. (a) Evaluate : 7.5

$$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$$

by change of order of integration.

(b) Using triple integral, find the volume of sphere : 7.5

$$x^2 + y^2 + z^2 = a^2$$

3. (a) Verify Stoke's theorem for $\vec{f} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by $x = \pm a$, $y = 0, y = b$. 7.5

(b) Verify divergence theorem for $\vec{f} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$. 7.5

UNIT – II

4. (a) Solve the differential equation : 7.5

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

(b) Solve by method of variation of parameters : 7.5

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

5. (a) Solve the differential equation : 7.5

$$p(p + y) = x(x + y)$$

(b) Solve the differential equation : 7.5

$$y - 2px = \tan^{-1}(xp)^2$$

UNIT – III

6. (a) Prove that $f(z) = \sin z$ is analytic and find its derivative. 7.5

(b) Find an analytic function whose real part is : 7.5

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

7. (a) Show that the function $f(z)$ defined by :

$$f(z) = \frac{x^2y^3(x + iy)}{x^6 + y^{10}};\ z \ne 0, f(0) = 0$$

is not analytic at origin although C–R equations are satisfied at origin. 7.5