Roll No. ....

OLE-3008

B. Tech. 1st Semester (CSE)
Examination – April, 2021

MATH - I (CALCULUS AND LINEAR ALGEBRA)

Paper : BSC-MATH-103-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. (a) Evaluate $\lim_{x\to 0}(2^x + x)^{1/x}$. $2.5 \times 6 = 15$

(b) Give the physical significance of Rolle's Theorem.

(c) Show that the commutative law is not valid for the multiplication of matrices $A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.

(d) Find the matrix representation of linear map $T : R^2 \to R^2$ defined by :

$$T(x, y) = (3x + 4y, 2x - 5y)$$

relative to the usual basis $\{(1, 0), (0, 1)\}$.

(e) Define elementary matrix. Explain with the help of an example.

(f) If the eigen values of a matrix A are 1, 3, 4 then find the eigen values of A^{-1} and A^3.

UNIT – I

2. (a) Verify Cauchy's mean value theorem for the function $\sin x$ and $\cos x$ in the interval $[a, b]$ 7

(b) A given quantity of metal is to be cast into a half cylinder, i.e., with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter to the ends is $\pi/_{\pi+2}$. 8

3. (a) Find the volume formed by the revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$, about the x-axis. 7

(b) Establish the relation between beta and gamma function. 8

UNIT – II

4. (a) Solve the following equations by Gauss elimination method : 7

$$2x + 4y - 6z = -4$$
$$x + 5y + 3z = 10$$
$$x + 3y + 2z = 5$$

(b) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ by reducing it to its normal form. 8

5. (a) Show that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ 7

(b) Find for what values of a the system of linear equations : 8

$$2x - 3y + 6z - 5w = 3$$

$$y - 4z + w = 1$$

$$4x - 5y + 8z - 9w = a$$

has (i) no solution (ii) infinite number of solutions. In case of infinite many solutions, find the general solution of given system of linear equations.

UNIT – III

6. (a) Show that the vectors $v_1 = (1, 0, -1)$, $v_2 = (1, 2, 1)$, $v_3 = (0, -3, 2)$ form a basis for R^3. Express standard basis vector (0, 1, 0) as a linear combination of v_1, v_2, v_3. 7

(b) Prove that the set $R^{m \times n}$ of all $m \times n$ matrices over the field R of real numbers is a vector space over the field R of real numbers with respect to the addition of matrices as vector addition and multiplication of matrix by a scalar as scalar multiplication. 8

7. (a) Verify rank nullity theorem for the linear transformation $T : R^3 \to R^3$ defined by : 9

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$$

(b) Let U, V and W are vector spaces over the same field K. Prove that for the linear transformations, F : U → V and G : V → W, composite function G°F is a linear map from U to W. 6

UNIT – IV

8. (a) Prove that every orthogonal set of non-zero vectors in an inner product space is linearly independent. 7

(b) Use Gram-Schmidt orthogonalization process to construct an orthonormal basis of R^3 for the vectors : 8

$$u_1 = (1, -1, 0); u_2 = (2, -1, -2), u_3 = (1, -1, -2)$$

9. (a) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Also diagonalize the given matrix A. 10

(b) If P and Q are orthogonal matrices, prove that PQ is also orthogonal. 5